WEEKLY TEST TYJ-02 SOLUTION 01 AUGUST 2019

MATHEMATICS

31. (b) $x = 2 + 2^{2/3} + 2^{1/3} \Rightarrow x - 2 = 2^{2/3} + 2^{1/3}$

Cubing both sides, we get

$x^3 - 8 - 6x^2 + 12x = 6 + 6(x - 2)$

$\Rightarrow x^3 - 6x^2 + 6x = 2$.

32. d) Given that $x^{2/3} - 7x^{1/3} + 10 = 0$. Given equation can be written as $(x^{1/3})^2 - 7(x^{1/3}) + 10 = 0$

Let $a = x^{1/3}$, then it reduces to the equation

$a^2 - 7a + 10 = 0 \Rightarrow (a-5)(a-2) = 0 \Rightarrow a = 5, 2$

Putting these values, we have $a^3 = x \Rightarrow x = 125$ and 8.

33. (d) Given $|x|^2 - 3|x| + 2 = 0$

Here we consider two cases *viz.* $x < 0$ and $x > 0$

Case I : $x < 0$ This gives $x^2 + 3x + 2 = 0$

$\Rightarrow (x+2)(x+1) = 0 \Rightarrow x = -2, -1$

Also $x = -1, -2$ satisfy $x < 0$, so $x = -1, -2$ is solution in this case.

Case II : $x > 0$. This gives $x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0 \Rightarrow x = 2, 1$, so $x = 2, 1$ is solution in this case. Hence the number of solutions are four i.e. $x = -1, 1, 2, -2$

Aliter : $|x|^2 - 3|x| + 2 = 0$

$\Rightarrow (|x| - 1)(|x| - 2) = 0$

$\Rightarrow |x| = 1$ and $|x| = 2 \Rightarrow x = \pm 1, x = \pm 2$.

34. (c) Given equation is $(p-q)x^2 + (q-r)x + (r-p) = 0$

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow x = \frac{(r-q) \pm (q+r-2p)}{2(p-q)} \Rightarrow x = \frac{r-p}{p-q}, 1$$

35. (d) If $x \neq 1$, multiplying each term by $(x-1)$, the given equation reduces to $x(x-1) = (x-1)$ or $(x-1)^2 = 0$ or $x = 1$, which is not possible as considering $x \neq 1$. Thus given equation has no roots.

36. (d) $x + \frac{1}{x} = 2 \Rightarrow x + \frac{1}{x} - 2 = 0 \ (\because x \neq 0)$

$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1, 1$.

37. (a) According to condition $\frac{\alpha^2 + 1}{2} = -\alpha$

$\Rightarrow \alpha^2 + 2\alpha + 1 = 0 \Rightarrow \alpha = -1, -1$.

38. b) Let $x = 2 + \cfrac{1}{2 + \cfrac{1}{2 + ...\infty}}$

$\Rightarrow x = 2 + \frac{1}{x}$ (on simplification)

$\Rightarrow x = 1 \pm \sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1 + \sqrt{2}$ is required answer.

39. (d) The equation $(|x| - 4)(|x| - 3) = 0$

$\Rightarrow |x| = 4 \Rightarrow x = \pm 4 \quad \Rightarrow |x| = 3 \Rightarrow x = \pm 3$.

40. (a) We have $x = \sqrt{7 + 4\sqrt{3}}$

$$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{7+4\sqrt{3}}} = \frac{\sqrt{7-4\sqrt{3}}}{\sqrt{7+4\sqrt{3}}.\sqrt{7-4\sqrt{3}}}$$

$$= \sqrt{7 - 4\sqrt{3}}$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}$$

$$= (\sqrt{3} + 2) + (2 - \sqrt{3}) = 4 .$$

41. (a) Given $\sqrt{(x+1)} - \sqrt{(x-1)} = \sqrt{(4x-1)}$

Squaring both sides, we get $-2\sqrt{(x^2 - 1)} = 2x - 1$

Squaring again, we get $x = 5/4$ which does not satisfy the given equation. Hence equation has no solution

42. (d) $x^2 + 5|x| + 4 = 0 \Rightarrow |x|^2 + 5|x| + 4 = 0$

$\Rightarrow |x| = -1, -4$, which is not possible. Hence, the given equation has no real root.

43. (b) Let roots are α and $-\alpha$, then sum of the roots

$$\alpha + (-\alpha) = \frac{3(\lambda - 2)}{2} \Rightarrow 0 = \frac{3}{2}(\lambda - 2) \Rightarrow \lambda = 2$$

44. (d) Equation $2x^2 - kx + x + 8 = 0$ has equal and real roots, then $D = b^2 - 4ac = 0$.

$\Rightarrow (1-k)^2 - 4.2.8 = 0 \Rightarrow k^2 + 1 - 2k - 64 = 0$

$\Rightarrow k^2 - 2k - 63 = 0 \Rightarrow k = 9, -7$.

45. (c) Roots of $x^2 - 8x + (a^2 - 6a) = 0$ are real. So $D \geq 0$

$\Rightarrow 64 - 4(a^2 - 6a) \geq 0 \Rightarrow 16 - a^2 + 6a \geq 0$

$\Rightarrow a^2 - 6a - 16 \leq 0 \Rightarrow (a-8)(a+2) \leq 0$

Now we have two cases:

Case I : $(a-8) \leq 0$ and $(a+2) \geq 0$

$\Rightarrow a \leq 8$ and $a \geq -2$

Case II : $(a-8) \geq 0$ and $(a+2) \leq 0$

$\Rightarrow a \geq 8$ and $a \leq -2$ but it is impossible

Therefore, we get $-2 \leq a \leq 8$

Aliter : Students should note that the expression $(x-a)(x-b)\{a < b\}$ will be less than or equal to zero if $x \in [a, b]$ or otherwise $x \notin [a, b]$.

Therefore $(a-8)(a+2) \leq 0$

i.e., $\{a - (-2)\}(a - 8) \leq 0 \Rightarrow a \in [-2, 8]$.